Waves - III

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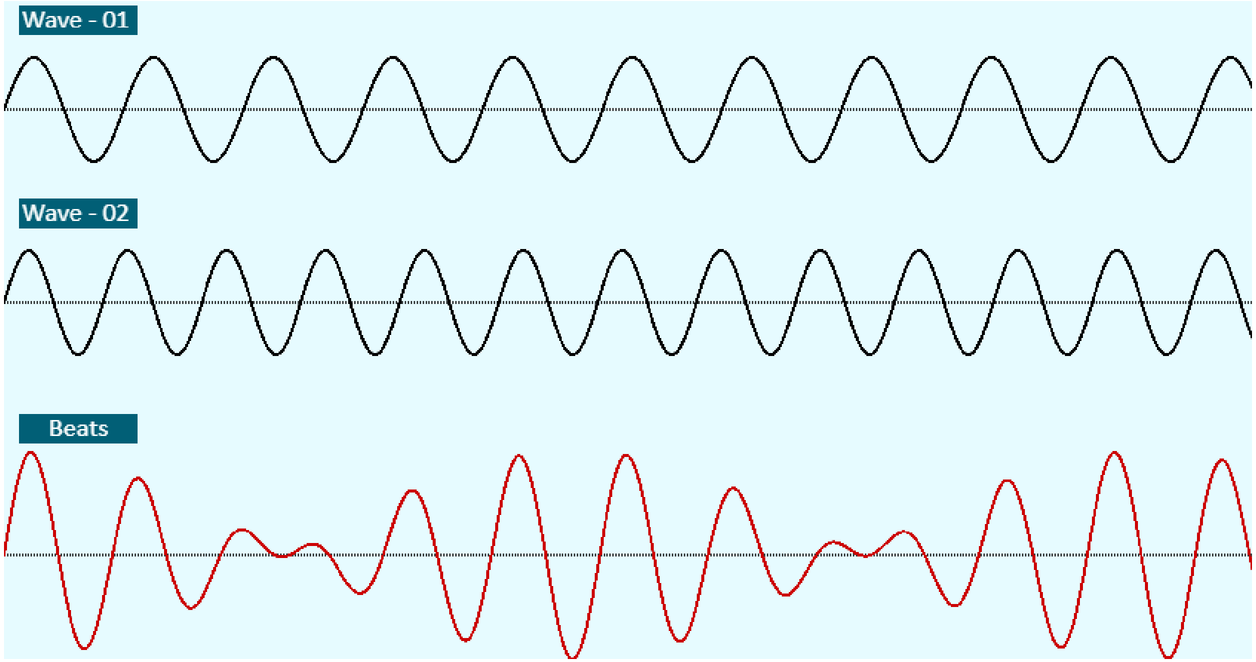
- Beats
- Echo
- Doppler effect

Beats

When two progressive waves of difference frequencies, propagating in the same direction, superimpose each other, it results in waxing and waning of sound. This phenomenon is called beats.



Click here for simulation

Uses of beats : for tuning musical instruments, for determination of frequency of tuning fork

Beats (mathematical development)

Let the equations of the two progressive waves be

$$y_1 = A\sin \omega_1\left(t - \frac{x}{v}\right) \qquad y_2 = A\sin \omega_2\left(t - \frac{x}{v}\right)$$

Using principle of superposition, their resultant is given by

$$y_{res} = 2A\sin\left(\frac{\omega_1 + \omega_2}{2}\right)\left(t - \frac{x}{v}\right)\cos\left(\frac{\omega_1 - \omega_2}{2}\right)\left(t - \frac{x}{v}\right)$$

denoting $\left(\frac{\omega_1 + \omega_2}{2}\right)$ as ω and $(\omega_1 - \omega_2)$ as $\Delta\omega$

$$y_{res} = 2A\cos\left[\left(\frac{\Delta\omega}{2}\right)\left(t - \frac{x}{v}\right)\right]\sin\left[\omega\left(t - \frac{x}{v}\right)\right]$$

Number of beats "heard" per second (called beat frequency) is given by

$$\boxed{\Delta n = |\, n_1 - n_2 \,|}$$

Echo

It is a phenomenon of reflection of sound in which the initial and the reflected sound are heard distinctly.

Least distance for a distinct echo is 16.5 m

Two sound can be heard distinctly if the time interval between them is more than persistence of hearing i.e. 0.1 s.

Time taken by the sound, generated by the person, to undergo reflection and reach the person is given by

$$t = \frac{2L}{v}$$

Considering the speed of sound in air as 330 ms^{-1} and persistence of hearing we get

$$\frac{2L}{330} \geq 0.1$$

$$\Rightarrow \quad L \geq \frac{33}{2}$$

$$\Rightarrow \quad L \geq 16.5 \text{ m}$$

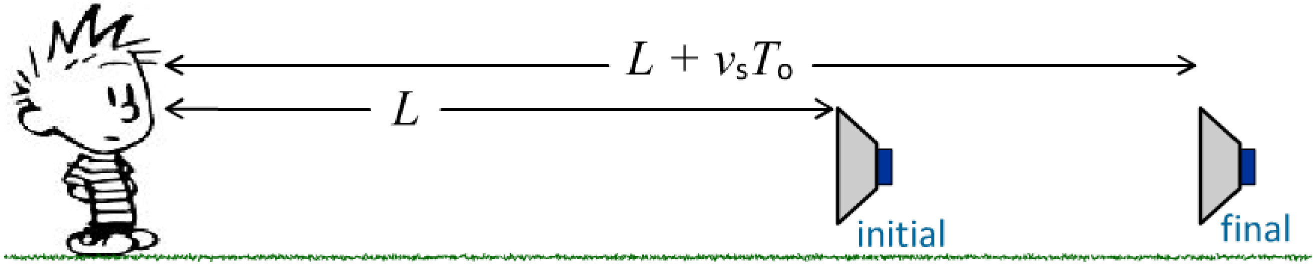
Doppler effect

It is an apparent change in the frequency observed due to the relative motion between the source and the observer.

Different cases

- Source moving away from a stationary observer
- Source moving towards a stationary observer
- Observer moving away from a stationary source
- Observer moving towards a stationary source
- Observer and source moving towards each other
- Observer and source moving away from each other
- Observer and source moving in the same direction with source in front of the observer

Source moving away from a stationary observer



Time taken by a pulse, emitted at t = 0, to reach the observer is

$$t_1 = \frac{L}{v}$$

Distance between source and observer after time interval of T_o is

$$S = L + v_s T_o$$

Time at which this pulse reaches the observer (from this position is)

$$t_2 = T_o + \frac{L + v_s T_o}{v}$$

Time in which (m+1)th pulse reaches the observer (from its initial position is)

$$t_{m+1} = mT_o + \frac{L + v_s m T_o}{v}$$

Time in which m pulses are heard by the observer is

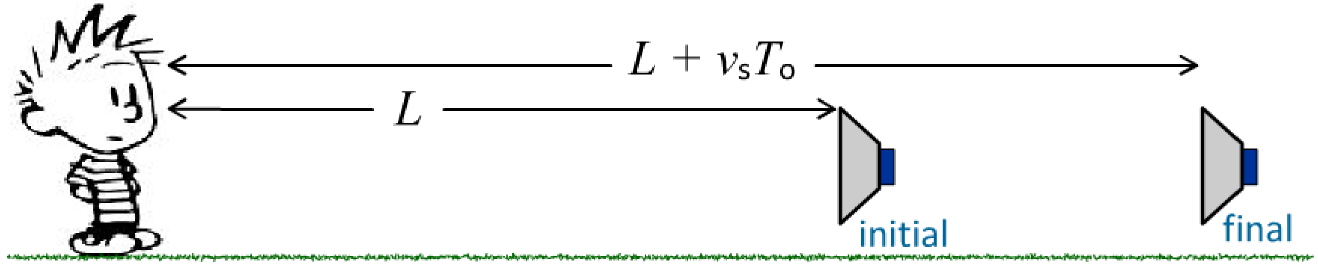
$$t = mT_o + \frac{L + v_s m T_o}{v} - \frac{L}{v}$$

$$t = mT_o + \frac{v_s m T_o}{m}$$

Time interval between each pulse (t/m) is

$$T_{app} = T_o + \frac{v_s T_o}{v}$$

Source moving away from a stationary observer



$$T_{app} = T_o\left[1 + \frac{v_s}{v}\right]$$

Frequency of observed pulses is

$$n_{app} = n_o\left[1 + \frac{v_s}{v}\right]^{-1}$$

$$n_{app} = n_o\left[\frac{v}{v + v_s}\right]$$

Apparent frequency (as heard by the observer) is <u>*less*</u> than the frequency of sound emitted

Source moving towards a stationary observer

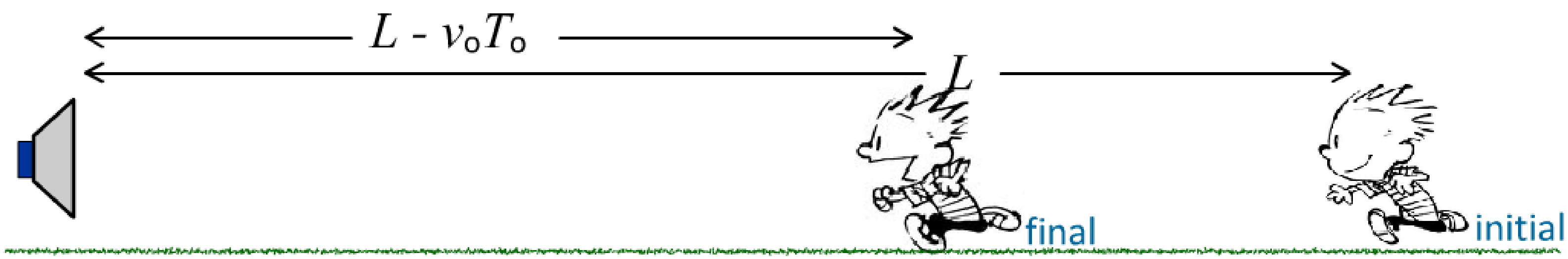
Replacing v_s with $-v_s$ we get

$$n_{app} = n_o\left[\frac{v}{v + (-v_s)}\right]$$

$$n_{app} = n_o\left[\frac{v}{v - v_s}\right]$$

Apparent frequency (as heard by the observer) is <u>*more*</u> than the frequency of sound emitted

Observer moving towards the stationary source



Working in the frame of reference of moving observer, time interval between the first and the (m+1)th pulse to reach the observer is

$$t_{m+1} - t_1 = mT_o - \frac{v_o m T_o}{v_o + v}$$

Therefore observed time period is

$$T_{app} = T_o\left[1 - \frac{v_o}{v_o + v}\right]$$

$$T_{app} = T_o\left[\frac{v}{v_o + v}\right]$$

Apparent frequency due to Doppler effect is given by

$$n_{app} = n_o\left[\frac{v_o + v}{v}\right]$$

Causes of changes in frequency of a source

- When a tuning fork is loaded with wax, its frequency decreases
- When a tuning fork is filed, its frequency increases
- When length of a vibrating segment is increased its frequency decreases
- When the tension is stretched string is increased its frequency increases
- When the length of an open pipe or a closed pipe is increases the frequency decreases
- When distance between the source and observer increases, the apparent frequency heard by the observer decreases.
- When distance between the source and observer decreases, the apparent frequency heard by the observer increases.

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